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Analytical and Experimental Study on Ethernet Passive Optical Networks: Challenges and Solutions

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Abstract: *Ethernet passive optical networks* (EPONs) have emerged as a promising solution for the next generation access networks. As this technology matures, intensive research work is underway to enhance its functional capability and economic viability. In this work we review some existing MAC control protocols designed for EPON system. Then we study on the performance characteristic of an EPON model with both analytical and experimental approaches. By investigating the temporal relationship of multiple events successively occurred in the system, a graphical presentation is developed to facilitate more quantitative analysis. Based on classical queuing theory, we derive closed-form expression for the average packet queuing delay and average queue length of the researched EPON model. Simulation experiments show that the derived analytical expression can precisely evaluate the network performance for memoryless traffic inputs, as well as to closely estimate the performance of lightly loaded network for bursty traffic profiles.

Keywords: Ethernet passive optical network; media access control; resource allocation; modeling and analysis; simulation and performance evaluation.

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1 Introduction

Ethernet passive optical network is viewed by many as a promising solution for the next generation access network. With optical speed Ethernet frames, maintenance-less *passive optical network* (PON) architecture, as well as newly standardized *quality-of-service* (QoS) functionalities, EPON is capable of delivering bundled voice and data services along with video broadcast over the same high-speed infrastructure in a cost-effective way.

A PON is basically an *optical line terminal* (OLT) residing in the *central office* (CO) connected to multiple *optical network units* (ONUs) near subscribers' premises. In this configuration, all ONUs share a single landline running between the CO and a passive optical splitter/coupler located near the service neighborhood. Fig. 1 illustrates the key elements of an EPON setup. Due to the practical cost constraint of access networks, most EPON applications deploy one optical fibre for both downstream and upstream transmissions and separate channels of two directions by different carrier wavelengths (1550nm for downstream and 1310nm for upstream). Particularly, packets in the downstream direction are broadcast by the OLT and received by every ONU. Each ONU selectively extracts packets containing its unique media access address. Therefore, the downstream transmission is point-to-multipoint and is well suited to the broadcast nature of Ethernet. In the upstream transmission, on the contrary, multiple ONUs share the same point-to-point transmission channel from the optical splitter/coupler to the OLT. Therefore, an appropriate access control protocol is required to avoid transmission collision. Since the optical splitter/coupler is designed for unidirectional transmission and precludes communication between different ONUs, most distributed scheduling based *media access control* (MAC) protocols such as CSMA/CD (*Carrier Sense Multiple Access with Collision Detection*) are inappropriate[a]. Consequently, centralized scheduling based *time division multiple access* (TDMA) schemes have gained major attentions both from research community and from industrial applications of EPON technology. In centralized scheduling based TDMA schemes, during a transmission cycle each ONU is assigned a time slot in which the ONU bursts its backlogged traffic into the uplink channel. This time slot may be assigned by fixed allocation, or be granted dynamically based on polling, where in the latter case the duration of a transmission opportunity may change cycle-by-cycle and may be different from one ONU to another. Due to the non-uniform behavior of traffic generated in *local area networks* (LANs), *dynamical bandwidth allocation* (DBA) is viewed as more bandwidth-efficient and exhibits better performance over static allocation schemes Kramer (Feb. 2002); Assi (2003). DBA is currently a quite popular research topic of EPON technology and has been extensively studied by many researchers. In this study we first briefly survey some notable MAC control protocols of EPON system. Then we evaluate the statistical performance metrics of an EPON system that applies centralized TDMA scheduling and DBA for upstream transmission. Particularly, we draw analytical conclusions on the system behavior with well-modeled

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[a] A new modification of the existing EPON architecture that allows application of distributed scheduling is proposed in Foh (2004). This design requires an additional local optical fibre between the splitter/coupler and each ONU, to enable the upstream transmissions being echoed back to every ONU. Here one transmission cycle is divided into a contention-based "request period" during which sources bid to be first in the token-passing list, and a "data period" whereby data packets are transmitted.

memoryless traffic inputs, and envision its performance by experimental method for more realistic bursty traffic profiles.

The rest of this paper is organized as follows. Section 2 introduces related background knowledge and summarizes several MAC control protocols with different design considerations. Section 3 exposits our proposed analytical framework on the studied EPON system. Section 4 compares the mathematical analysis with simulation and visualizes the deviation of our analysis for memoryless traffic inputs from the performance driven by bursty traffic sources. Finally, Section 5 presents concluding remarks on this study.

2 Background knowledge and existing EPON MAC control protocols

2.1 MPCP based EPON MAC operations

In order to enable efficient statistical bandwidth multiplexing in EPON architecture, the IEEE 802.3ah has devised the *multipoint control protocol* (MPCP) IEEE 802.3ah (2004). MPCP defines a message-based control mechanism to facilitate real-time information exchange between the MAC peers at the OLT and each ONU. Two messages are involved in the regular operation mode of EPONs, i.e., *REPORT* message and *GATE* message. The *REPORT* message is sent by each active ONU and received by the OLT, to update the OLT's perception on the amount of buffered traffic at the ONU. The *GATE* message is sent by the OLT and received by the destined ONU, to inform the ONU with its granted transmission window size and upcoming transmission start time. The assignment of transmission window is based on the polling result provided by the *REPORT* message of each ONU. According to the DBA schemes studied in the literature, this transmission window can be assigned in different ways. For example, it could be assigned by:

*A*1 *Gated service scheme*—the granted transmission window for polling cycle n exactly matches the requested window by the ONU in polling cycle $n-1$, i.e., $G_i(n) = R_i(n-1)$, where $G_i(m)$ and $R_i(m)$ are the granted window size to ONU i and the requested window size by ONU i in cycle m, respectively.

*A*2 *Limited service scheme*—the granted transmission window is upper-bounded at a fixed value W_{max} with no over-granting, i.e., $G_i(n) = \min\,[R_i(n-1),\, W_{max}]$.

*A*3 *Credit-based service schemes*—the granted transmission window is upper-bounded at W_{max} and permits certain amount of over-granting above the requested window by the ONU, i.e., $G_i(n) = \min\,[R_i(n-1) + \delta,\, W_{max}]$. The value of δ can be a constant value or be proportional to the ONU's request, i.e., $\delta = \alpha R_i(n-1)$, where α is a design parameter.

*A*4 *Non-guaranteed elastic service scheme*—the granted transmission window is not constrained by W_{max} but depends on the total amount of grants issued by the most recent $N-1$ *GATE* messages when $G_i(n)$ is computed, denoted as $\phi(i)$, i.e., $G_i(n) = \min\,[R_i(n-1),\, N \times W_{max} - \phi(i)]$, where N is the number of active ONUs in the network. Thus, it is possible that $G_i(n) = N \times W_{max}$ if $\phi(i) = 0$, in which case however, the next $N-1$ *GATE* messages should not grant any bandwidth for upstream data transmission. Therefore, this scheme does not provide guaranteed upstream channel access to the ONUs.

$A5$ *Guaranteed elastic service scheme*—the granted transmission window to each ONU is first guaranteed by W_{max}. Above this guaranteed grant, ONUs requesting larger transmission window than W_{max} can share the available resource spared by other ONUs requesting less than W_{max}, i.e.,

$$G_i(n) = \begin{cases} R_i(n-1), & R_i(n-1) \leq W_{max} \\ W_{max} + \nu_i(n), & R_i(n-1) > W_{max} \end{cases} \tag{1}$$

where $\nu_i(n)$ is the extra allocation issued to heavily loaded ONU i by sharing available resource from lightly loaded ONUs.

The polling operation is also diverse in the literature. In general, it can be categorized into three classes as Zheng (2005):

$P1$ *Poll-and-stop polling*—the ONUs are polled and allowed to transmit one after the other, with a complete round-trip message walking overhead time required for each ONU. This scheme provides the most up-to-date bandwidth need information to the OLT before each ONU is allowed to transmit. However, as we can see, this scheme requires large signaling overhead withdrawn from the scarce network resource, especially when the OLT is serving a large number of ONUs.

$P2$ *Interleaved polling with stop*—the polling messages, i.e., *GATE* messages, for each ONU are broadcast sequentially to the downlink at the beginning of each service cycle, i.e., a service round where each ONU is serviced one time. Upon receiving and processing these polling messages, each ONU is allowed to transmit before the polling response (*REPORT*) message of the previous ONU arrives at the OLT. This scheme permits the *GATE* messages concurrently pass through the downlink channel and uplink data burst to be concatenated with the previous *REPORT* message in the uplink channel, thereby to reduce signaling overhead and shrink the service cycle. However, the initiation of *GATE* messages in service cycle n is contingent on the complete reception of a *REPORT* message from each ONU in service cycle $n-1$.

$P3$ *Interleaved polling*—the *GATE* message of service cycle n for each ONU is initiated by the OLT, upon receiving the *REPORT* message of service cycle $n-1$ only from this ONU. This scheme allows the downlink *GATE* messages and uplink data bursts as well as corresponding *REPORT* messages to coexist in the signal propagation "pipe". Therefore, it offers the most efficient usage of the wavelength resource among the three polling schemes. Nevertheless, compared with $P2$, this scheme may force the OLT to make suboptimal decision on resource allocation without considering demand information from all ONUs, due to the lack of "stop" occurred in $P2$.

More detailed graphical illustration of the control message flows in these different polling operations is given by Fig. 4 in Zheng (2005).

Since distances between the OLT and ONUs may vary, the data bursts from different ONUs may collide if the transmission of each ONU is scheduled with the same time reference. To avoid this collision, the standard defines *ranging* process, where the OLT updates its perception on each ONU's round trip propagation time via every *GATE-REPORT* message exchange with the ONU IEEE 802.3ah (2004). Being aware of the message propagation time to each ONU, the OLT therefore can adjust the scheduling of each ONU's transmission start time accordingly, such that data bursts from unevenly distanced ONUs are ultimately lined up without collision at the receiver of the OLT.

In order to effectively delimit data bursts from neighborly scheduled transmission windows, a guard time is defined before the start of each transmission window. This guard time permits an instant break, thereby the transmitters of ONUs can be completely turned on/off, and the OLT can adjust its receiving power threshold to detect signals from unevenly distanced ONUs.

2.2 Existing EPON MAC control protocols

Many of the existing research works of EPON technology focus on the DBA design and transmission scheduling algorithms. Below we briefly introduce some notable research driven by different design motivations. Readers may also find other good summarizations of this topic in McGarry (2004) and McGarry (2006).

2.2.1 MAC protocols using IPACT

Kramer (Feb. 2002) addressed many important issues on the DBA design of EPON system and initialized the interleaved polling operation. In this paper the authors proposed an *interleaved polling with adaptive cycle time* (IPACT) scheme. Alongside interleaved polling, i.e., $P3$ discussed above, multiple DBA schemes are proposed and quantitatively evaluated. Particularly, coupled with $P3$, DBA schemes $A2$, $A3$ and $A4$ as detailed in Section 2.1 are tested by simulation, in terms of average packet service delay. With the interleaved polling messages and flexible service cycle time, IPACT accommodates bursty traffic profile with efficient bandwidth usage. The paper concludes that using IPACT the *limited service* scheme $A2$ offers the best delay performance among the tested schemes. Nevertheless, IPACT is a *single service class* (SSC) based design framework, where packets are serviced on *first come first serve* (FCFS) basis without prioritization.

2.2.2 MAC protocols supporting differentiated services

Following IPACT, other service-centric and more sophisticated DBA schemes and scheduling design are proposed in the literature. In Kramer (Aug. 2002) the authors extended the SSC based design into *multiple service class* (MSC) case, where in each ONU multiple FCFS queues with different service priorities feed the shared uplink channel. The authors examined the limited service scheme with interleaved polling for MSC, i.e., $A2 + P3 + MSC$, and noticed the *light load penalty* problem. This is caused by the strict priority outbound transmission scheduling policy applied at the intra-ONU level. In order to better utilize the leftover bandwidth from ONUs with smaller traffic backlogs in the limited service scheme, Assi (2003) proposed DBA1 and DBA2 schemes in which ONU nodes are partitioned into two groups, i.e., underloaded and overloaded, according to their minimum guaranteed transmission window sizes. This work applies *guaranteed elastic service* scheme for

MSC, i.e., $A5 + MSC$. In DBA1 the polling operation follows $P2$. Since DBA2 improves DBA1 by allowing the *GATE* messages of underloaded ONUs for the next service cycle to be released before all *REPORT* messages in the current service cycle arrive, the polling operation by DBA2 can be viewed as a hybrid version of $P2$ and $P3$. In Xie (2004), the authors presented a new perspective of DBA, i.e., two-layer DBA (TLBA), where the total available bandwidth is allocated among different classes first, then among ONUs for intra-class allocation. In this TLBA scheme, each service class is assigned a weight to guarantee it a minimum bandwidth allocation and to prevent bandwidth monopoly by high priority classes. In this way, each ONU is guaranteed a minimum transmission window, however, conditioned on its instantaneous class-level bandwidth need. Since TLBA requires the OLT to receive all *REPORT* messages from ONUs in order to calculate the total demand of each service class, the polling operation is *interleaved polling with stop*. Namely, this TLBA applies $A5 + P2 + MSC$. Moreover, to improve the jitter performance of high priority voice service, Shami (2005) proposed a *hybrid granting protocol* (HGP). The idea of HGP is to prevent the delay of voice packets from severe variation due to the service fluctuation of other classes. This is achieved by assigning the high priority class into a separate service sub-cycle within a complete service cycle. Furthermore, the bandwidth need of this class is predicted by the OLT and is allocated in *grant-before-report* (GBR) manner, to reduce extra service delay entailed by polling. The scheduling for medium and low priority classes by HGP is based on $A5 + P2 + MSC$. Finally, Ma (2005) proposed a *bandwidth guaranteed polling* (BGP) design, where the ONUs are partitioned into *bandwidth guaranteed* (BG) and *non-bandwidth guaranteed* (non-BG) groups to support ONU-level service differentiation. Here the uplink capacity is divided into multiple entries of equal size. By pre-assigning each BG ONU corresponding number of entries, its minimum service opportunity is guaranteed. The remaining entries and the idled portion of any BG ONU's entry is assigned to non-BG ONUs on "best effort" basis. Since in BGP ONUs are polled one after the other and each grant is upperbounded by the size of a capacity entry, this scheme can be categorized as $A2 + P1 + SSC$.

2.2.3 MAC protocols with fairness control

In Kramer (2004) a *fair queuing with service envelopes* (FQSE) algorithm for the MAC scheduling of EPON is proposed and analyzed. FQSE targets on fully centralized scheduling at the OLT for each priority queue in the ONUs. This requires the ONU to request bandwidth separately for each of its housed priority queue, which may not be possible if the number of priority queues in an ONU is large (*REPORT* message can only carry up to eight bandwidth requests). Therefore, in this case FQSE requires the ONU to send an estimated piece-wise linear service envelope to the OLT. This service envelope enables the OLT to ensure both *sibling fairness* as well as *cousin fairness* for bandwidth allocation Kramer (2004). Besides, the fairness issue in EPON MAC operation is also quantitatively addressed in Bai (2006) by employing the concept of *fairness index*, along with the proposed *weight-based DBA* (WDBA) scheme. This WDBA aims to enhance *inter-ONU statistical bandwidth multiplexing* (T-SBM), by maintaining a maximum fairness index during the process of bandwidth allocation by the OLT. The design of FQSE and WDBA are both based on $A5 + P2 + MSC$.

2.2.4 MAC protocols with admission control

Admission control (AC) in EPON system is a new area gaining considerable research efforts. The general principle of AC is to accept a new service flow only if the requested bandwidth of the incoming flow can be guaranteed while the committed service of existing flows is not compromised. Assi (2007) proposed an AC framework in EPON system, where the uplink capacity is divided into three components, with one for *best effort* traffic AC at the OLT and two for global and local AC of real-time service flows performed by the OLT and each ONU respectively. Coupled with this AC design, an *admission-control-enabled DBA* (AC-DBA) scheme is proposed to specify the DBA operation for admitted traffic flows. This AC-DBA scheme defines a *super-cycle* and employs a *crediting system* to ensure that long-term bandwidth provisioning of admitted flows is guaranteed while short-term bandwidth under-utilization is avoided. According to the taxonomy depicted in Section 2.1, this MAC scheduling approach also falls into $A5 + P2 + MSC$.

3 Analytical study on EPON

3.1 Existing EPON analytical research

Most of the above research works use simulations or minor mathematical calculations to evaluate the designed network performance. Currently, only a few works in the literature have provided mathematical analysis on the performance of EPONs. Specifically, based on the proposed BGP protocol, Ma (2005) developed a mathematical model to roughly estimate the delay performance offered by BGP. However, one limitation of this analysis is that it omits the inherent correlation between the length of one service cycle and the amount of data transmitted in the subsequent service cycle, taking the arrival process into consideration. Another good analytical study of EPON is proposed in Bhatia (2006). Here the authors developed a recursive model for the IPACT scheme suggested in Kramer (Feb. 2002). Utilizing this recursive model the authors derived closed-form expression of average queue size at the ONU for single-ONU and multiple-ONU network setups. For single-ONU case as well as the case of multiple-ONU with small load-distance ratio, where the latter case can be understood as multiple single-ONU networks operating concurrently, the analytical model can be categorized as $A1 + P1 + SSC$. For the case of multiple-ONU with large load-distance ratio, the analysis falls into $A1 + P3 + SSC$. However, the paper stated that in this case, the analytical model starts showing error when non-uniform traffic input such as Poisson or self-similar traffic is applied. The reason is that the constant bit arrival rate applied here effectively leads to a fixed service cycle for the analyzed model. When non-uniform arrival occurs, the service cycle length tends to vary, due to the correlation between one service cycle length and data amount transmitted in subsequent service cycle, as we have mentioned before.

3.2 Proposed analytical model

Now it is well understood that modeling EPON system with variable service cycle length by investigating aforementioned correlation is trivial and yet cumbersome. In this study we present some insights on the performance of EPON as a special case of polling system. We view an EPON as a complete functional module that delays the proceeding of an arriving packet, and model its performance based on classical approaches for queuing analysis. The derived analytical framework can precisely model the performance of EPON with non-uniform Poisson traffic inputs and estimate the performance when the network is lightly-loaded with self-similar traffic inputs. Specifically, we consider a SSC based EPON system applying *interleaved polling with stop* operation and *gated service* scheme for bandwidth allocation, i.e., $A1 + P2 + SSC$, and study on the delay performance of such system. To form the model, we apply the following assumptions:

1. Packet arrivals to each ONU's MAC buffer follow Poisson distribution, with the average arrival rate of $\lambda/m\, packets/second$, where λ and m are the average arrival rate to the entire network and the number of ONUs in the network, respectively. Namely, the network load is equally distributed over each ONU.
2. Packet size is independent from the packet arrival process and is uniformly distributed between 64 and 1518 bytes, i.e., the minimum and maximum size of an Ethernet frame.
3. The network is not overloaded and therefore a steady state exists.
4. The buffer size at each ONU is large enough to contain the backlogged traffic, i.e., no packet dropping.
5. ONUs are separated from the OLT with the same distance, which can be virtually achieved through the periodic *ranging* process as discussed in Section 2.1.
6. Message propagation time from the OLT to a certain ONU does not change with time[b].

In order to facilitate later discussion, we first employ a graphical presentation as shown in Fig. 2 to illustrate the sequential events occurred in one service cycle by the *interleaved polling with stop* operation $P2$. Here, we denote the ONU scheduled for the earliest transmission within a service cycle as ONU 1 and similarly, ONU scheduled for the second transmission as ONU 2, and so forth. As shown in the figure, the graph organizes the key events occurred in one service cycle into a circle that reveals the temporal relationship of these events. Specifically, at the beginning of a service cycle, the OLT sends each ONU a *GATE* message. Since we apply uniform OLT-ONU distance, i.e., equal message propagation time, the *GATE* message destined to ONU 1 is transmitted first to the downstream channel[c]. When ONU

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[b] In practical systems, this value may vary due to small deviation of fiber refractive index resulted from temperature drift Kramer (Aug. 2002).

[c] Note that if the OLT-ONU distances are various, some *GATE* messages destined to other ONUs should be transmitted ahead of the one destined to ONU 1. However, this only incurs a fixed amount of extra overhead time in Fig. 2 without more analytical complexity.

1's *GATE* message is transmitted, after the downlink message propagation time ONU 1 receives this message and processes it with corresponding message processing time. Followed by a guard time, as shown in Fig. 2, ONU 1 starts transmitting. When the granted bytes have been transmitted, the last portion of the transmission window is used to transmit the *REPORT* message. During the transmission of ONU 1, other ONUs are continuously receiving *GATE* messages broadcast through the downlink and processing the one destined to respective ONU. Since the time required for ONU 2 to receive its *GATE* message from the downlink equals the time for ONU 1 to transmit a *REPORT* message into the uplink (*GATE* message and *REPORT* message are both 64 bytes long), even if ONU 1 only transmits a *REPORT* message with no data packet in its granted transmission window, ONU 2 will not miss its transmission start time, which is informed by the second *GATE* message and scheduled a guard time later than the termination of ONU 1's transmission. Namely, the arrival of ONU 2's *GATE* message will never be late and entail extra overhead time between consecutive transmission windows. Hence, for the same reason, we can conclude that every ONU is ready for transmitting after the guard time preceding its scheduled transmission start time. Now immediately after the termination of ONU 1's transmission, followed by a guard time, ONU 2 starts transmitting, and so forth. When the ONU that is scheduled for the latest transmission in this service cycle, i.e., ONU m, has completed transmitting its *REPORT* message, the system has to wait for this message to propagate from ONU m to the OLT through the uplink channel. Upon receiving this *REPORT* message, a computation time is required for the OLT to finish processing all of the received messages from ONUs and to perform window assignment for the next service cycle. After this computation time, the OLT then sends *GATE* messages sequentially again into the downlink channel, to initiate the next service cycle.

We define the following terms in the graphical presentation of Fig. 2 to express our analysis more concisely:

1. α–gap interval, the aggregated amount of time consumed by a guard time and its preceding *REPORT* message transmission time;
2. β–propagation interval, the aggregated amount of time consumed by the round trip propagation time, OLT computation time, ONU 1's *GATE* message transmission time and ONU message processing time (see Fig. 2);
3. φ–the total overhead time involved in one service cycle, i.e., $\varphi = m\alpha + \beta$;
4. *Arrival period* (AP)–the arrival period of ONU i is the time duration consisting of ONU i's data transmission interval and the preceding gap interval (and the propagation interval for $i = 1$);
5. *Forward neighbor* (FN)–ONU i's j^{th} FN is ONU $i + j$ modulo m ($1 \leq i \leq m$ and $0 \leq j \leq m - 1$). For example, ONU 2 is the first FN of ONU 1 and the latter is the first FN of ONU m as well as the 0^{th} FN of itself.

3.2.1 Average packet queuing delay

Consider the case where an arbitrary packet ϵ arrives during ONU i's AP at its j^{th} FN. The queuing delay of this packet includes three components:

1. The remaining transmission time of the packet being serviced (but not in the queue) at ONU i, or the remaining time of the overhead portion in ONU i's AP, when packet ϵ arrives. Namely, this is the waiting time before the service of ONU i's head-of-line (HOL) packet in the queue can be started. We denote this part as R.

2. The transmission time of all packets that will be transmitted before packet ϵ, after the service of HOL packet in ONU i's queue is started. These packets may belong to any ONU in the system. We denote this part as T.

3. The total duration of overhead time (gap intervals and propagation intervals) that will occur before packet ϵ is transmitted. We denote this part as G.

Therefore, the queuing delay of packet ϵ, denoted as D, is represented by the following equation:

$$D = R + T + G \tag{2}$$

When the system reaches steady state, the expected value of packet ϵ's queuing delay is then given as:

$$E(D) = E(R) + E(T) + E(G) \tag{3}$$

where $E(\cdot)$ denotes the expected value of the operand, i.e., the ensemble average. Since the arrival process is ergodic, time average and ensemble average in this case are inter-changeable. We neglect this terminological difference in the following discussion.

In steady state, packet ϵ will see the same average number of packets queued in the system (not in service), both when it is enqueued and when it is dequeued. Since the expected number of packets arrived at the system during packet ϵ's queuing time is $\lambda E(D)$, the expected number of packets transmitted during T is also $\lambda E(D)$. If let $\overline{X}$ denote the average transmission time of a packet, noting the independency of arrived packet size from the arrival process, the expected value of T is then given as:

$$E(T) = \lambda \times E(D) \times \overline{X} = \rho E(D) \tag{4}$$

where $\rho = \lambda\overline{X}$ is the average traffic intensity offered to the system. Combining (3) and (4), the expected value of packet ϵ's queuing delay is computed as:

$$E(D) = \frac{1}{1-\rho}\left[E(R) + E(G)\right] \tag{5}$$

Now the evaluation of average packet queuing delay is translated into finding the expected value of R and G. The former can be derived using a modified approach for the analysis of $M/G/1$ system with vacations in Bertsekas (1992), as in our case there are multiple queues and also the vacation periods, i.e., the overhead portions in Fig. 2, are not allowed to appear continuously. Specifically, the value of R, as function of packet ϵ's arrival time t, i.e., $R(t)$, is illustrated in Fig. 3, where AP_i and x_k denote the arrival period of ONU i and the transmission time of the k^{th} packet serviced by the system, respectively. It is shown in the figure that the overhead portion (α or $\alpha + \beta$), i.e., vacation, alternates with the data transmission interval.

Moreover, the overhead portion of the same ONU's AP appears one time in every m vacations. Suppose that at time t_0 the system has reached steady state and an arrival period of ONU m, i.e., AP_m has just expired. Considering that triangles in Fig. 3 are all right-angled and isoceles, the average value of R for ONU i, denoted as $\overline{R_i}$, can be computed as:

$$\overline{R_i} = \frac{1}{w_i t_0}\left[\sum_{x_n \in S_i(t_0)} \frac{1}{2}x_n^2 + \frac{1}{2}v_i^2 L_i(t_0)\right] \tag{6}$$

where we denote w_i as the proportion of ONU i's APs occupying time interval $[0,\, t_0)$, v_i the length of vacation portion in ONU i's AP (α or $\alpha + \beta$), $L_i(t_0)$ the number of vacations appeared in ONU i's APs during time interval $[0,\, t_0)$, and $S_i(t_0)$ the set of packets serviced by ONU i during $[0,\, t_0)$, respectively. Since each ONU equally shares the offered traffic load and the packet size is independent from the arrival process, when t_0 increases, the number of packets serviced by ONU i during time interval $[0,\, t_0)$ can be more and more precisely estimated as $M(t_0)/m$, where $M(t_0)$ is the number of packets serviced by the system during time interval $[0,\, t_0)$. Moreover, the number of vacations appeared in ONU i's APs during $[0,\, t_0)$ is just $L(t_0)/m$, with $L(t_0)$ denoting the total number of vacations occurred during $[0,\, t_0)$, as these vacations appearing in ONU i's APs repeat once in every m consecutive vacations. Therefore, (6) can be represented as:

$$\begin{aligned}\overline{R_i} &= \frac{1}{w_i t_0}\left[\frac{1}{m}\sum_{k=1}^{M(t_0)} \frac{1}{2}x_k^2 + \frac{1}{2}v_i^2\frac{L(t_0)}{m}\right] \\ &= \frac{1}{2mw_i}\left[\frac{M(t_0)}{t_0}\frac{\sum_{k=1}^{M(t_0)} x_k^2}{M(t_0)} + v_i^2\frac{L(t_0)}{t_0}\right]\end{aligned} \tag{7}$$

Noting that $\lim_{t_0 \to \infty} \frac{M(t_0)}{t_0} = \lambda$ and $\lim_{t_0 \to \infty} L(t_0) = t_0(1-\rho)/\overline{v}$, where $\overline{v} = \varphi/m$ denotes the average duration of one vacation period and ρ is also the average uplink utilization, we can infer that when $t_0 \to \infty$, the steady state value of $\overline{R_i}$ is given as:

$$\overline{R_i} = \frac{1}{2mw_i}\left[\lambda\overline{x_k^2} + (1-\rho)\frac{v_i^2}{\overline{v}}\right] = \frac{1}{2w_i}\left[\frac{\lambda}{m}\overline{x_k^2} + (1-\rho)\frac{v_i^2}{\varphi}\right] \tag{8}$$

The average value of R, i.e, $E(R)$, over the entire system then can be obtained as:

$$E(R) = \sum_{i=1}^{m} p_i(\epsilon)\overline{R_i} \tag{9}$$

where $p_i(\epsilon)$ denotes the probability for packet ϵ to arrive during an AP of ONU i. Noting that when $t_0 \to \infty$ the value of w_i approaches to $p_i(\epsilon)$, (8) and (9) indicate that:

$$E(R) = \frac{1}{2}\left[\lambda\overline{X^2} + \frac{(1-\rho)}{\varphi}\sum_{i=1}^{m} v_i^2\right] \tag{10}$$

where we use $\overline{X^2} = \overline{x_k^2}$ to concisely represent the second order moment of packet transmission time. Considering that $v_i = \alpha + \beta$, $(i = 1)$ for AP_1 and $v_i = \alpha$, $(2 \le i \le m)$ for other $m - 1$ APs, the value of $E(R)$ is ended up with:

$$E(R) = \frac{1}{2}\left\{\lambda\overline{X^2} + \frac{(1-\rho)}{\varphi}\left[(m-1)\alpha^2 + (\alpha+\beta)^2\right]\right\} \tag{11}$$
$$= \frac{1}{2}\left[\lambda\overline{X^2} + (1-\rho)\frac{m\alpha^2 + \beta^2 + 2\alpha\beta}{\varphi}\right]$$

To find the value of $E(G)$ in (5), it is given by probability theory that the expected value of G can be obtained as:

$$E(G) = \sum_{i=1}^{m}\sum_{j=0}^{m-1} p_i^j(\epsilon)G_i^j(\epsilon) \tag{12}$$

where $G_i^j(\epsilon)$ is the value of G when packet ϵ arrives during ONU i's AP at its j^{th} FN, and $p_i^j(\epsilon)$ is the corresponding stationary probability when the system reaches steady state. Since the overhead portion in ONU 1's AP is longer than the one in other APs (see Fig. 2), the calculation of $E(G)$ is split into two parts, i.e.,

$$E(G) = \sum_{j=0}^{m-1} p_1^j(\epsilon)G_1^j(\epsilon) + \sum_{i=2}^{m}\sum_{j=0}^{m-1} p_i^j(\epsilon)G_i^j(\epsilon) \tag{13}$$

Considering that ρ is also the steady state link utilization, in Fig. 2 the time proportions occupied by data transmission intervals and overhead times are therefore ρ and $1 - \rho$, respectively. Given the memoryless property of Poisson arrivals, this equivalently indicates that the steady state probability that packet ϵ arrives during any data transmission interval is ρ and during any overhead time period is $(1 - \rho)$. Moreover, in steady state the length of data transmission interval in every AP shares the same average value, as ONUs are equally loaded. The probability that packet ϵ arrives during ONU i's AP, i.e, $p_i(\epsilon)$ is then given as:

$$p_1(\epsilon) = \frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho) \tag{14}$$

$$p_i(\epsilon) = \frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho) \qquad (2 \le i \le m) \tag{15}$$

Since each ONU shares $1/m$ proportion of the overall average arrival rate λ, whenever packet ϵ arrives, it belongs to every ONU with probability of $1/m$. Mathematically, this implies that the conditional probability that packet ϵ arrives at ONU i's j^{th} FN, given that it arrives during ONU i's AP, i.e., $p^j(\epsilon|i)(1 \le i \le m,\ 0 \le j \le m-1)$ is $1/m$. Combining with (14) and (15), the probability of $p_i^j(\epsilon)$ in (13) is resulted in:

$$p_1^j(\epsilon) = p^j(\epsilon|1)p_1(\epsilon) = \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho)\right] \qquad (0 \le j \le m-1) \tag{16}$$

$$p_i^j(\epsilon) = p^j(\epsilon|i)p_i(\epsilon) = \frac{1}{m}\left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho)\right] \qquad (2 \le i \le m,\ 0 \le j \le m-1) \tag{17}$$

The value of $G_i^j(\epsilon)$ in (13) is to be analyzed by two cases:

1. The j^{th} FN of ONU i is inclusively between ONU i and ONU m, i.e., $0 \leq j \leq m-i$. In this case after packet ϵ arrives, certain number of gap intervals will appear before its parent ONU is polled, whereby the transmission time of packet ϵ is requested. Namely, the gap interval immediately appears before the data transmission interval of ONU $i+1$, ONU $i+2$, until ONU $i+j$. Particularly, no gap interval will appear when $j=0$. Therefore, the amount of overhead time consumed by these gap intervals is $j\alpha$. Note that any arriving packet has first to wait until its transmission time is requested by the nearest upcoming *REPORT* message of the parent ONU. Then in the following transmission opportunity of the parent ONU, this packet can be ultimately transmitted.

2. The j^{th} FN of ONU i is inclusively between ONU 1 and ONU $i-1$, i.e., $m-i+1 \leq j \leq m-1$ (see Fig. 2). In this case after packet ϵ arrives, certain number of gap intervals and a propagation interval will appear before its parent ONU is polled, whereby the transmission time of packet ϵ is requested. Namely, the gap interval immediately appears before the data transmission interval of ONU $i+1$, until ONU m, then ONU 1 until ONU $i+j-m$, and the propagation interval appears before ONU 1's data transmission interval. Therefore, the amount of time consumed by these gap intervals and the propagation interval is $j\alpha+\beta$.

In both cases above, once the transmission time of packet ϵ is requested, an extra set of gap intervals and a propagation interval are required before the transmission of packet ϵ starts. This amount of time consumption is just φ, the total overhead time occurred in one service cycle. Therefore, the value of $G_i^j(\epsilon)$ in (13) is finally computed as:

$$G_i^j(\epsilon) = j\alpha + \varphi \qquad (1 \leq i \leq m,\ 0 \leq j \leq m-i) \tag{18}$$

$$G_i^j(\epsilon) = j\alpha + \beta + \varphi \qquad (1 \leq i \leq m,\ m-i+1 \leq j \leq m-1) \tag{19}$$

Collecting (16)–(19), (13) yields (20) as:

$$\begin{aligned} E(G) &= \sum_{j=0}^{m-1} p_1^j(\epsilon) G_1^j(\epsilon) + \sum_{i=2}^{m} \left[\sum_{j=0}^{m-i} p_i^j(\epsilon) G_i^j(\epsilon) + \sum_{j=m-i+1}^{m-1} p_i^j(\epsilon) G_i^j(\epsilon) \right] \\ &= \sum_{j=0}^{m-1} \frac{1}{m} \left[\frac{1}{m}\rho + \frac{\alpha+\beta}{\varphi}(1-\rho) \right] (j\alpha+\varphi) \\ &\quad + \sum_{i=2}^{m} \left\{ \sum_{j=0}^{m-i} \frac{1}{m} \left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho) \right] (j\alpha+\varphi) \right. \\ &\qquad \left. + \sum_{j=m-i+1}^{m-1} \frac{1}{m} \left[\frac{1}{m}\rho + \frac{\alpha}{\varphi}(1-\rho) \right] (j\alpha+\beta+\varphi) \right\} \end{aligned} \tag{20}$$

Substituting (11) and (20) into (5), with some simplifications, the steady state average packet queuing delay is ended up with:

$$E(D) = \frac{1}{2(1-\rho)}\left[\lambda\overline{X^2} + 3\varphi - \frac{\varphi}{m}\rho\right] \tag{21}$$

3.2.2 Average queue size

With the average packet queuing delay derived in (21), the steady state average queue length, i.e., average number of packets staying in the queue of one ONU, denoted as $E(N)$, is given by Little's theorem as Bertsekas (1992):

$$E(N) = \frac{\lambda}{m}E(D) = \frac{\lambda}{2m(1-\rho)}\left[\lambda\overline{X^2} + 3\varphi - \frac{\varphi}{m}\rho\right] \tag{22}$$

4 Simulation experiments

In this section we first verify the analysis proposed above by running Monte Carlo simulations. Then we apply more bursty self-similar traffic profiles to the studied EPON system and compare the experimental outcomes with the analytical results based on Poisson traffic inputs.

We developed a simulation environment by *ns-2* GNU (1989), to verify the above analysis. The relevant parameters used for the simulation are listed in Table 1. We have neglected the OLT computation time and ONU message processing time, as they do not qualitatively affect our analysis. In the simulation, we sampled the number of queued packets, i.e., queue length, at a tagged ONU every 2ms, to compare with the value obtained through theoretical derivation.

Fig. 4 and Fig. 5 illustrate the average packet queuing delay performance by 0.1 second and 10 seconds simulations, respectively. We can see the perfect match of the simulated curve with corresponding theoretical calculation, especially for long enough simulation time, e.g., 10 seconds or longer. If the simulation time is not long enough, e.g., 0.1 second, it is shown in Fig. 4 that the delay curve measured by simulation constantly falls below, instead of fluctuating around, its theoretical counterpart, especially for high loading points. This is raised by the queue-building-up behavior at each ONU. Namely, before the system reaches steady state, packets will see shorter average queue length upon arrival and thereby exhibit smaller queuing delay, compared with the long-term measurement. This behavior tends to be more prominent for high loading points, as larger steady-state queue requires longer build-up period. The mis-match between transient and steady-state performances is more explicitly revealed by the average queue length in Fig. 6. Nevertheless, the precise evaluation of steady-state performance by (21) and (22) is verified by Fig. 5 and Fig. 7.

The correctness of our analytical model largely depends on the memoryless assumption of the offered traffic pattern. However, it is well known that traffic passing through access networks is bursty and possesses *long range dependency* (LRD) property Taqqu (1997). In our experiments, we also simulated the performance of the system with self-similar traffic inputs. Without claiming theoretical certainty, moreover, we compare this result with the derived analytical model, to visualize the confidence interval for applying our analysis over real network scenario. Table 2 includes the values of *Hurst parameter* ($\mathcal{H}$) measured in the simulation[d]. Fig. 8 and Fig. 9 illustrate the difference between simulation outcome and our analysis, in terms of average packet queuing delay and average queue length. We can see that for light loading scenarios, our analysis for memoryless traffic profiles is also able to estimate the network performance for bursty traffic inputs ($\mathcal{H} < 0.7$), with small deviation. For example, when the network is loaded with less than 60% of its uplink capacity, the deviation can be limited within 40%.

5 Conclusion

In this study, we first introduced some MAC control protocols designed for EPON system. Then we analyzed the performance of an EPON network applying *interleaved polling with stop* operation and *gated service* scheme for resource allocation. A graphical representation was formed to investigate the temporal behavior of such a system. We also derived closed-form expressions to evaluate the average packet queuing delay and average queue length of the researched EPON model. Simulation verified that this analytical framework can precisely evaluate the network performance for memoryless traffic inputs, and is capable of estimating the system performance with small deviation, for light loads of bursty traffic.

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[d] For each value of ρ, the value of $\mathcal{H}$ shown in the table is the averaged value of Hurst parameters for 16 traffic traces feeding the network. The Hurst parameter of each traffic trace is estimated by *least square approximation* and the measurement scale varies from 0.0625ms to 1ms.

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Table 1 Simulation Parameters

number of OLT	1
number of ONU	16
uplink capacity	1 *Gbps*
OLT-ONU distance	20 *km*
guard time	1 μs
REPORT message size	64 *bytes*
GATE message size	64 *bytes*
data packet size	uniformly $\in$ [64, 1518] *bytes*
OLT computation time	*neglected*
ONU message processing time	*neglected*

Table 2 Measured Hurst Parameters in Simulation

(ρ, $\mathcal{H}$)	(0.0498, 0.7367)	(0.1012, 0.7325)
(0.1495, 0.7450)	(0.1988, 0.7518)	(0.2506, 0.7623)
(0.2976, 0.7832)	(0.3445, 0.7871)	(0.3942, 0.7440)
(0.4475, 0.7715)	(0.4917, 0.7708)	(0.5541, 0.7592)
(0.6002, 0.7693)	(0.6360, 0.7532)	(0.6841, 0.7725)
(0.7529, 0.7830)	(0.7976, 0.7764)	(0.8485, 0.7661)
(0.8920, 0.7567)	(0.9375, 0.7601)	

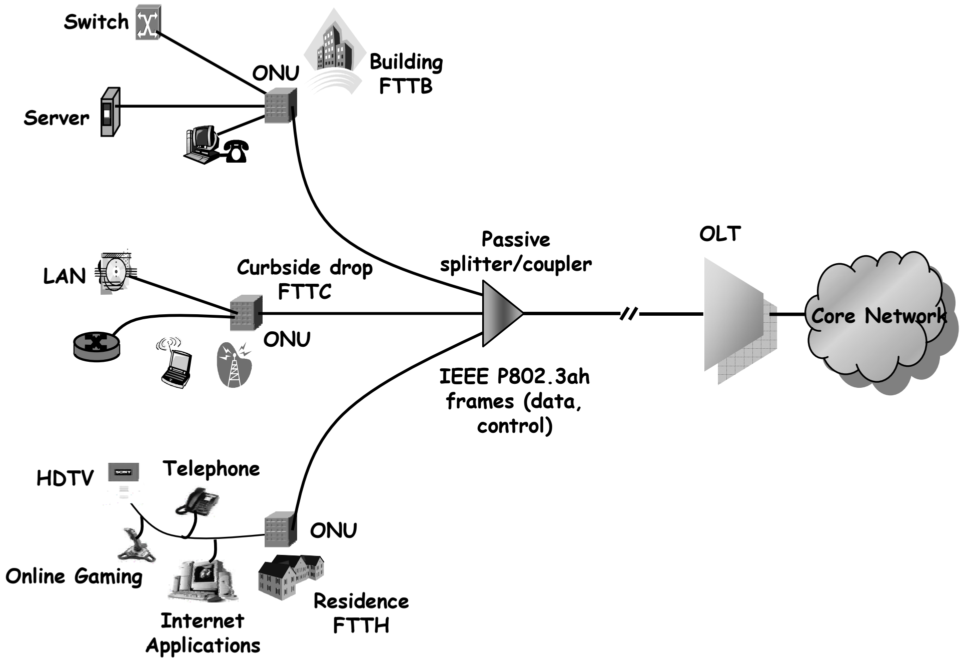

Figure 1 Illustration of EPON architecture

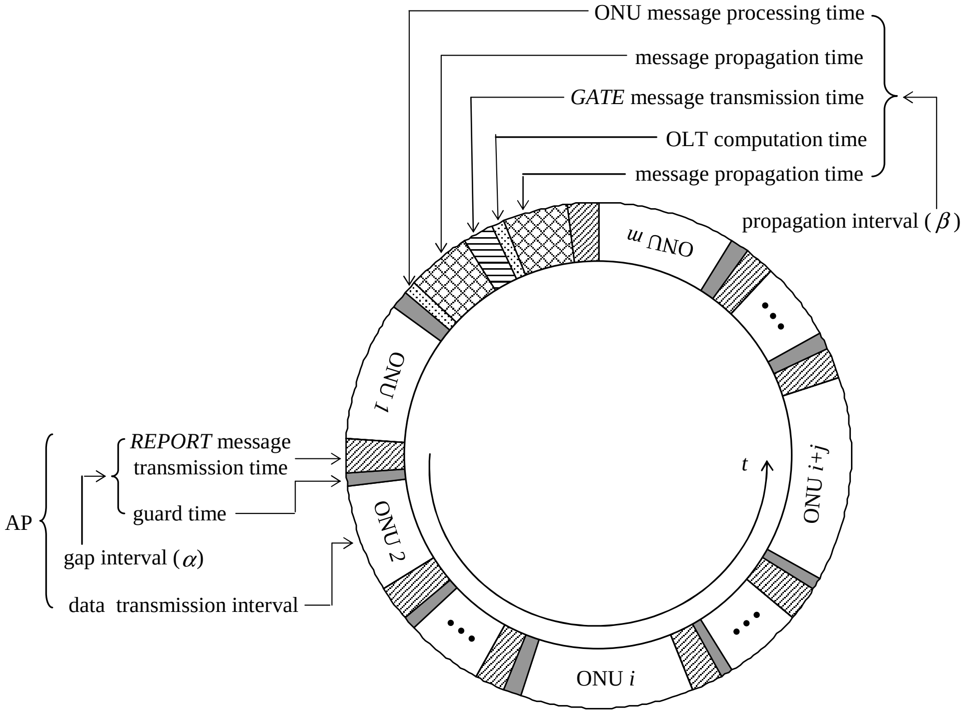

Figure 2 Graphical presentation of sequential events in one service cycle

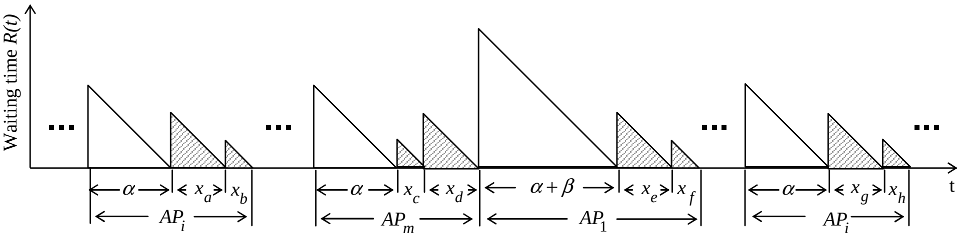

Figure 3 Illustration of waiting time as function of packet arrival time t

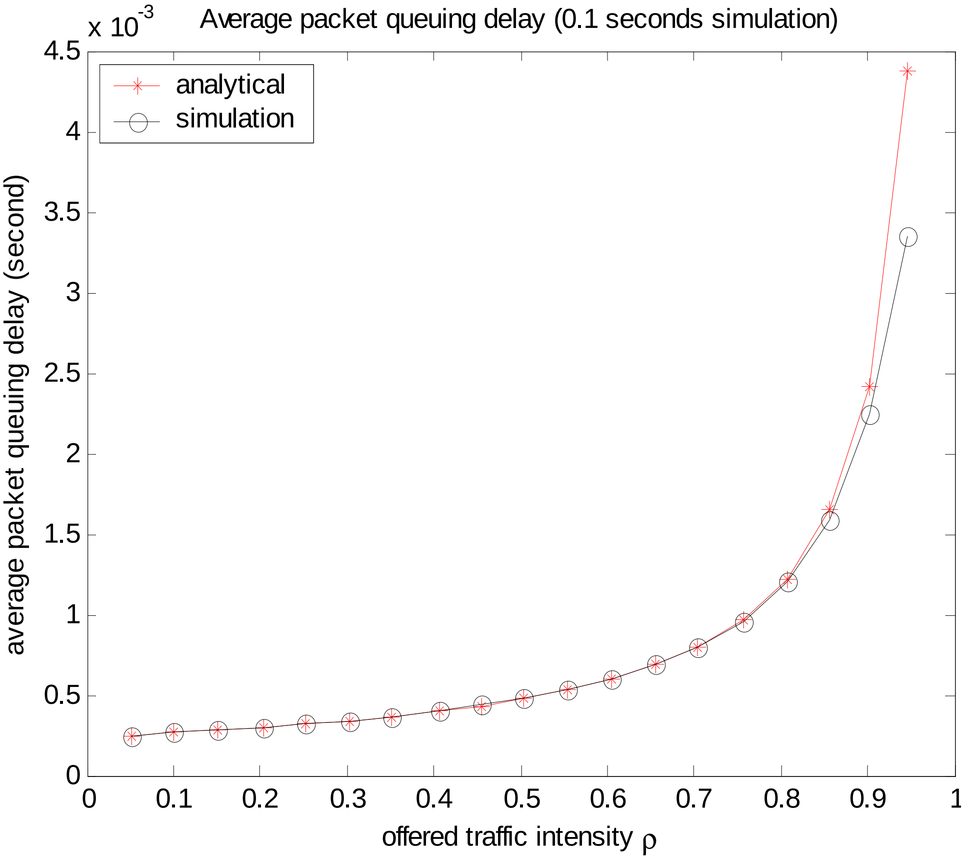

Figure 4 Average packet queuing delay by 0.1 second simulation (Poisson traffic inputs).

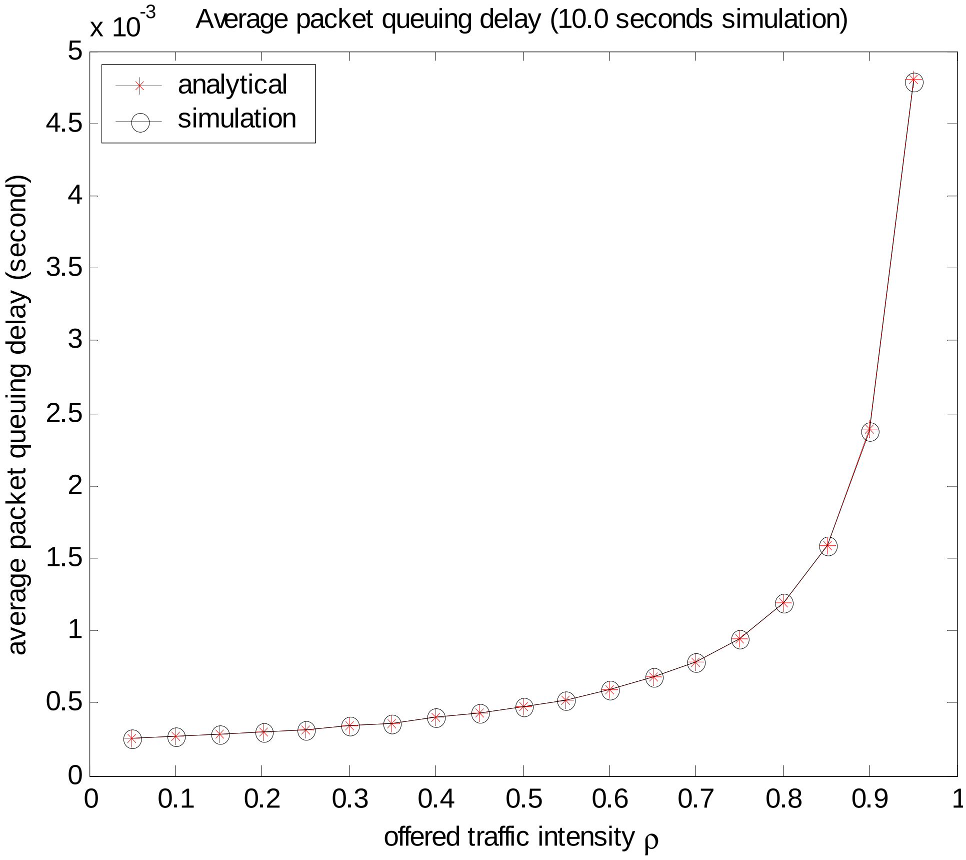

Figure 5 Average packet queuing delay by 10.0 seconds simulation (Poisson traffic inputs).

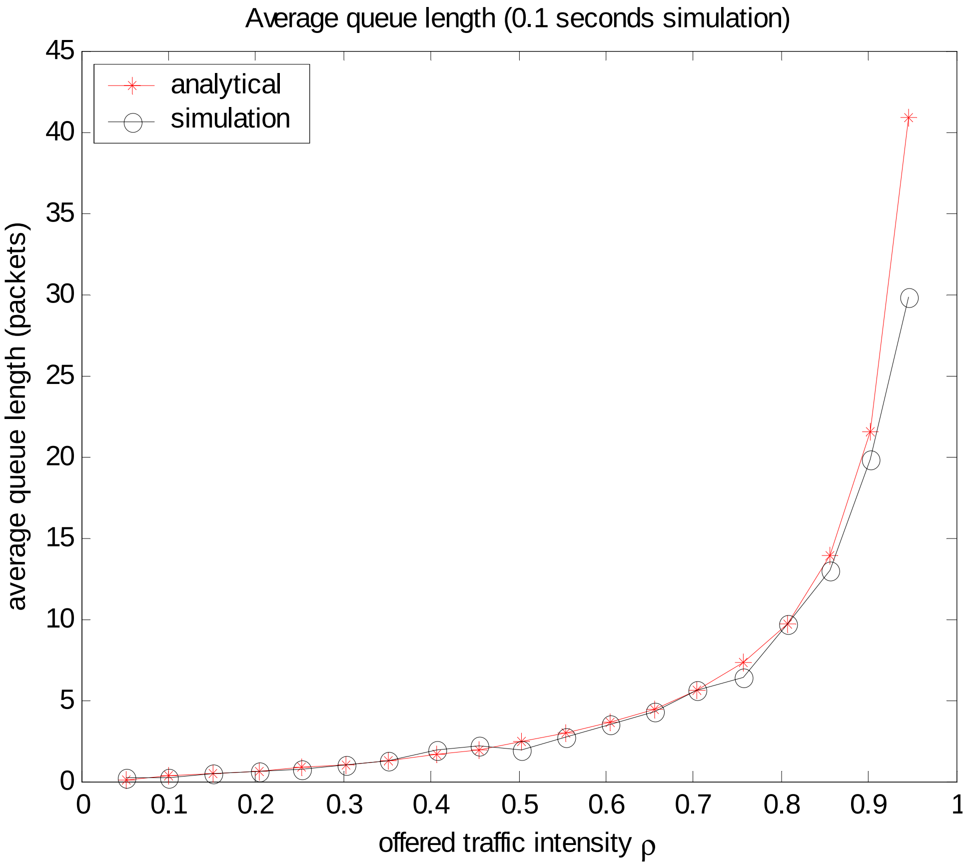

Figure 6 Average queue length per ONU by 0.1 second simulation (Poisson traffic inputs).

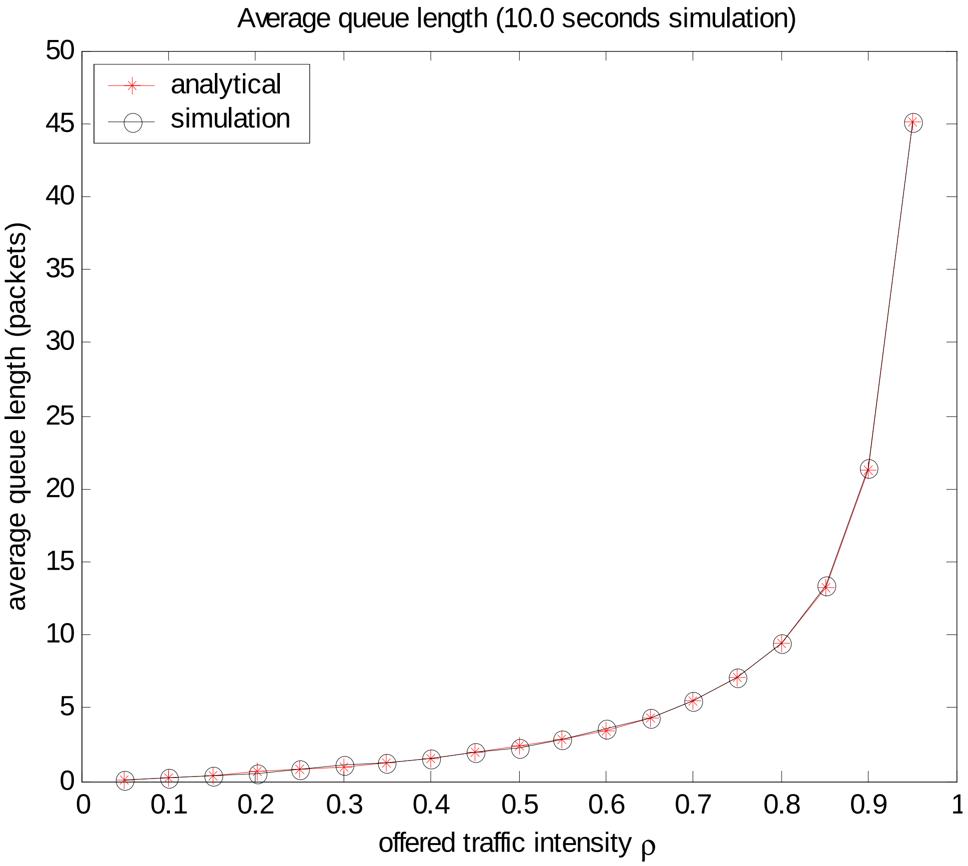

Figure 7 Average queue length per ONU by 10.0 seconds simulation (Poisson traffic inputs).

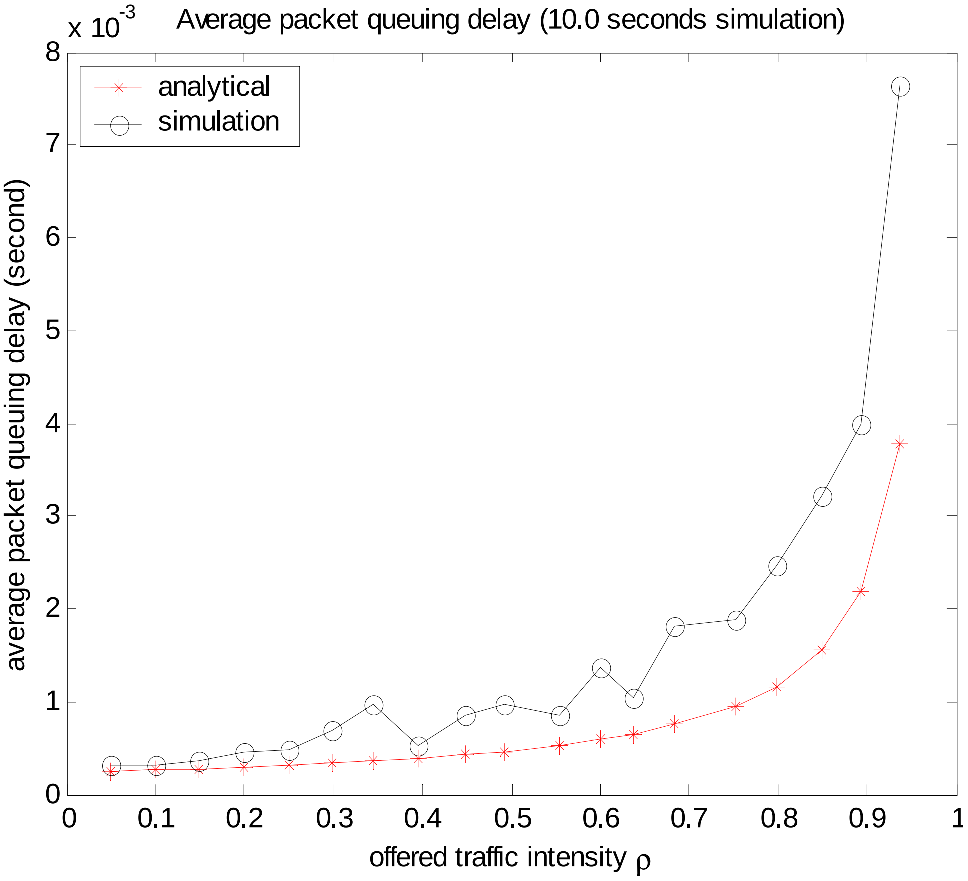

Figure 8 Average packet queuing delay by 10.0 seconds simulation (self-similar traffic inputs).

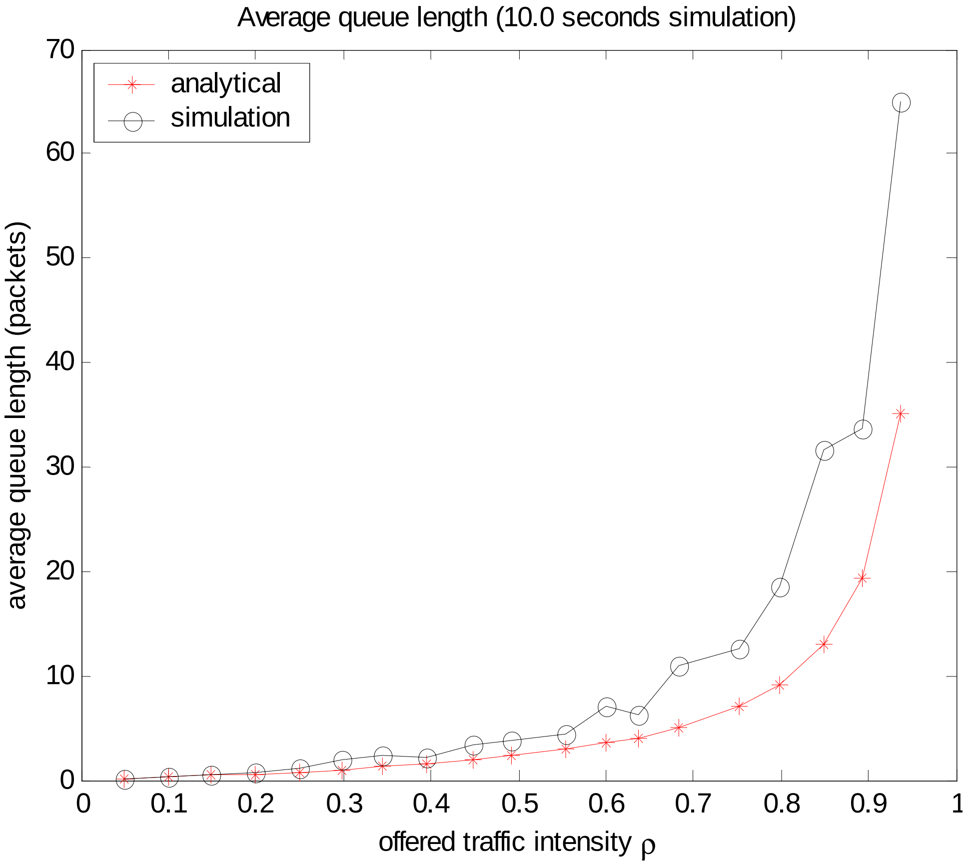

Figure 9 Average queue length per ONU by 10.0 seconds simulation (self-similar traffic inputs).